Electronic supplement, Crameri & Kaus:

1. 1-D code description.

In the 1-D code we consider a lithosphere compressed under constant strain rate such that 1-D and laterally homogeneous 2-D results are directly comparable. The equations describing visco-elasto-plastic rheology and the temperature evolution with shear heating are 1-D versions of the 2-D equations used in Burg and Schmalholz (2008).
The numerical solution for the temperature field is derived using a second order conservative finite difference scheme from the 1-D heat equation for variable conductivity (k), internal heating (H), shear heating (SH), heat capacity (c_p), density (ρ) and velocity (v_z) with depth and given by

$$\rho c_p \left(\frac{\partial T}{\partial t} + v_z \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial z} \left(k \frac{\partial T}{\partial z} \right) + H + SH.$$

Using the temperature field, the strength profile is derived by calculating visco-elastic trial stresses given as

$$\sigma_{xx}^{tr} = P_{tot} + \tau_{xx} \ and \ \sigma_{zz}^{tr} = P_{tot} + \tau_{zz},$$

where the total pressure is

$$P_{tot} = \rho g z - \frac{\tau_{xx} + \tau_{zz}}{2},$$

$$\tau_{xx} = \frac{2\eta_{eff}\dot{\varepsilon}_{xx} G \Delta t + \tau_{xx}^{old}}{G\Delta t + \eta_{eff}}, \tau_{zz} = \frac{2\eta_{eff}\dot{\varepsilon}_{zz} G \Delta t + \tau_{zz}^{old}}{G\Delta t + \eta_{eff}},$$

with the density ρ, the depth z, the time step Δt and the elastic shear modulus G. These stresses are subsequently corrected using a plastic yield criterion given by a Mohr-Coulomb yield function written as
$F = \tau^* - \sigma^* \sin(\phi) - c\cos(\phi)$,
where $F > 0$ is the yield criterion, ϕ is the friction angle, c the cohesion of the rocks. The radius (τ^*) and the centre (σ^*) of the Mohr-circle are given by

$$\tau^* = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{zz}}{2} \right)^2} and \ \sigma^* = -\frac{\sigma_{xx} + \sigma_{zz}}{2}.$$

If yielding occurs, the stress state at the given point is returned to the yield envelope: the plastic stress increments ($\Delta\sigma_{xx}^{pl}, \Delta\sigma_{zz}^{pl}$), which set a given stress state ($\sigma_{xx}^{tr}, \sigma_{zz}^{tr}$) outside the yield surface $F(\sigma^{tr}) > 0$ back to the yield surface ($\sigma_{xx}^{y}, \sigma_{zz}^{y}$) are given by

$$\Delta\sigma_{xx}^{pl} = -(1-f)\left(\frac{\sigma_{xx}^{tr} - \sigma_{zz}^{tr}}{2} \right),$$

$$\Delta\sigma_{zz}^{pl} = (1-f)\left(\frac{\sigma_{xx}^{tr} - \sigma_{zz}^{tr}}{2} \right),$$

where

$$f = \frac{-\left(\frac{\sigma_{xx}^{tr} + \sigma_{zz}^{tr}}{2} \right)\sin(\phi) + c\cos(\phi)}{\sqrt{\left(\frac{\sigma_{xx}^{tr} - \sigma_{zz}^{tr}}{2} \right)^2}},$$

with ϕ as the friction angle. Final stresses are given by

$$\sigma_{xx}^{y} = \sigma_{xx}^{tr} + \Delta\sigma_{xx}^{pl} \ and \ \sigma_{zz}^{y} = \sigma_{zz}^{tr} + \Delta\sigma_{zz}^{pl}.$$

At last, the second invariant of stress is given by

$$\sigma_{2nd} = \sqrt{\frac{\tau_{xx}^2 + \tau_{zz}^2}{2}},$$

where

$$\tau_{xx} = \sigma_{xx}^{y} + P, \ \tau_{zz} = \sigma_{zz}^{y} + P \ and \ P = -\frac{\sigma_{xx}^{y} + \sigma_{zz}^{y}}{2}.$$

The crust has Mohr-Coulomb plasticity, whereas low temperature plasticity is applied for the upper mantle and only for stresses higher than 200 MPa. The limit due to Peierls plasticity is calculated as

$$\eta_{Peierls} = \frac{\sigma_0}{\dot{\varepsilon}_{2nd}\sqrt{3}} \left(1 - \sqrt{\frac{RT}{H_0} \ln\left(\frac{\sqrt{3}\dot{\varepsilon}_0}{2\dot{\varepsilon}_{2nd}} \right)} \right),$$

where $\dot{\varepsilon}_{2nd}$ is the second invariant of the strain rate, T the temperature and values used are $\dot{\varepsilon}_0 = 5.7 \cdot 10^{11}$ s^{-1}, $\sigma_0 = 8.5 \cdot 10^9$ Pa *and* $H_0 = 525$ kJmol^{-1}. The effective viscosity η_{eff} is computed according to Equation 2 in the manuscript and subsequently corrected by $\eta_{eff} = \min[\eta_{eff}, \eta_{Peierls}]$.

Shear heating is calculated as

$$SH = \tau_{xx}\left(\dot{\varepsilon}_{xx} - \dot{\varepsilon}_{xx}^{el} \right) + \tau_{zz}\left(\dot{\varepsilon}_{zz} - \dot{\varepsilon}_{zz}^{el} \right),$$

where $\dot{\varepsilon}_{zz} = -\dot{\varepsilon}_{xx}$ and the elastic strain rate is

$$\dot{\varepsilon}_{xx}^{el} = \frac{1}{2G}\frac{\partial \tau_{xx}}{\partial t} = \frac{1}{2G}\frac{\tau_{xx}^{new} - \tau_{xx}^{old}}{\Delta t},$$

$$\dot{\varepsilon}_{zz}^{el} = \frac{1}{2G}\frac{\partial \tau_{zz}}{\partial t} = \frac{1}{2G}\frac{\tau_{zz}^{new} - \tau_{zz}^{old}}{\Delta t}.$$

τ_{xx} and τ_{zz} are the deviatoric stresses given by
$\tau_{xx} = \sigma_{xx} + P$ and $\tau_{zz} = \sigma_{zz} + P$.
The characteristic length scale L is continuously computed through time as the distance between the minimum and the maximum depth, where Peierls plasticity is applied and eventually extended by the brittle thickness of the lower crust in cases where the lower crust immediately above the Moho behaves brittle. The localization parameter Lo is thus also time dependent and indicates localization occurrence once it reaches $Lo > 1$. Both crust and lithosphere thicken with progressive shortening with a velocity $v_z = z \cdot \dot{\varepsilon}_{xx}$ resulting in a new depth $z_{new} = z_{old} - v_z\Delta t$.

Temperature is continuously calculated and coupled with the effective viscosity and the shear heating term. The 1-D code is benchmarked against analytical solutions for the temperature field and against the 2-D code.

2. Parameters.

Parameters used in this work are given in Table 1.

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Table 1. Parameters used in this work.

Parameter and Variables	Symbol	Value	Units
Standard model[1]			
Length		1000	km
Depth		120 [120-660]*	km
Granitic layer thickness (upper crust)		25	km
Diabase layer thickness (lower crust)		10 [0-55]*	km
Olivine layer thickness (upper mantle)		85 [85-625]*	km
Thermal expansion coefficient	α	$3.2 \cdot 10^{-5}$	K^{-1}
Heat capacity	c_p	1050	$Jkg^{-1}K^{-1}$
Cohesion	c	$1.0 \cdot 10^{7}$	Pa
Elastic shear modulus	G	$1.0 \cdot 10^{10}$	Pa
Friction angle	ϕ	30	$^\circ$
Peierls plasticity[2]			
	$\dot{\varepsilon}_0$	$5.7 \cdot 10^{11}$	s^{-1}
	σ_n	$8.5 \cdot 10^{9}$	Pa
	H_0	525	kJ mol^{-1}
Upper crust: dry granite[3]			
Density ($T = 0$ °C)	ρ_0	2700	kg m^{-3}
Power law exponent	n	3.3	
Radioactive heat production	H	$1.4 \cdot 10^{-6}$	W m^{-3}
Conductivity	k	2.5	W m^{-1} K^{-1}
Viscosity coefficient	A	$3.16 \cdot 10^{-26}$	Pa^{-n} s^{-1}
Activation energy	E	$1.9 \cdot 10^{5}$	J mol^{-1}
Weak lower crust: diabase[3]			
Density ($T = 0$ °C)	ρ_0	2900	kg m^{-3}
Power law exponent	n	3	
Radioactive heat production	H	$0.4 \cdot 10^{-6}$	W m^{-3}
Conductivity	k	2.1	W m^{-1} K^{-1}
Viscosity coefficient	A	$3.2 \cdot 10^{-20}$	Pa^{-n} s^{-1}
Activation energy	E	$2.76 \cdot 10^{5}$	J mol^{-1}
Upper mantle: wet olivine[3]			
Density ($T = 0$ °C)	ρ_0	3300	kg m^{-3}
Power law exponent	n	4	
Radioactive heat production	H	0	W m^{-3}
Conductivity	k	3.0	W m^{-1} K^{-1}
Viscosity coefficient	A	$2.0 \cdot 10^{-21}$	Pa^{-n} s^{-1}
Activation energy	E	$4.71 \cdot 10^{5}$	J mol^{-1}
Strong lower crust: Columbia diabase[3]			
Density ($T = 0$ °C)	ρ_0	2900	kg m^{-3}
Power law exponent	n	4.7	
Radioactive heat production	H	$0.4 \cdot 10^{-6}$	W m^{-3}
Conductivity	k	2.1	W m^{-1} K^{-1}
Viscosity coefficient	A	$1.2 \cdot 10^{-26}$	Pa^{-n} s^{-1}
Activation energy	E	$4.85 \cdot 10^{5}$	J mol^{-1}
Mars&Venus crust: Columbia diabase[3]			
Density ($T = 0$ °C)	ρ_0	2700	kg m^{-3}
Power law exponent	n	4.7	
Radioactive heat production	H	0	W m^{-3}
Conductivity	k	2.5	W m^{-1} K^{-1}
Viscosity coefficient	A	$1.2 \cdot 10^{-26}$	Pa^{-n} s^{-1}
Activation energy	E	$4.85 \cdot 10^{5}$	J mol^{-1}
Mars&Venus mantle: dry olivine[4]			
Density ($T = 0$ °C)	ρ_0	3300	kg m^{-3}
Power law exponent	n	3.5	
Radioactive heat production	H	0	W m^{-3}
Conductivity	k	3.0	W m^{-1} K^{-1}
Viscosity coefficient	A	$4.85 \cdot 10^{-17}$	Pa^{-n} s^{-1}
Activation energy	E	$5.35 \cdot 10^{5}$	J mol^{-1}

*values between brackets indicate the tested range of parameters
[1]denoted are initial values after e.g. [Carter and Tsenn 1987; Ranalli 1995]
[2][Goetze and Evans, 1979; Molnar and Jones, 2004]
[3][Mackwell et al., 1998; Afonso and Ranalli, 2004]
[4][Hirth and Kohlstedt, 1996]